Equation of lines & planes in $\mathbb{R}^3$.

1. Lines

start at one point

- base point + add a scalar multiples of a.
- direction vector

Ex) $(2,0,0) + t(0,3,4)$ as t ranges over all scalars, gives a line.

Parametric Equations for a line

The standard parametrization of the line through $P_0 = (x_0, y_0, z_0)$ parallel to $V = V_1 i + V_2 j + V_3 k$

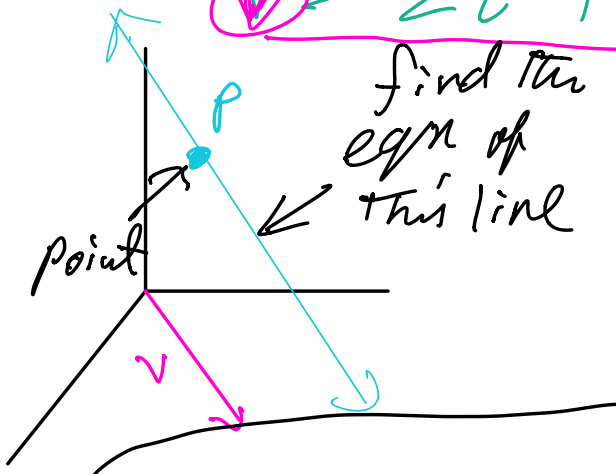
is $x = x_0 + tV_1$

$$y = y_0 + tv_2$$

$$z = z_0 + tv_3$$

Ex) Find the parametric eqn. of the line through $(-2, 0, 4)$ & parallel to

$$\vec{v} = 2\vec{i} + 4\vec{j} - 2\vec{k}$$



$$(-2+2t)\vec{i} + 4t\vec{j} + (4-2t)\vec{k}, \quad t \in \mathbb{R}$$

Ex) parametrization the line through $(2, 4, 6)$ & $(3, 4, 8)$

base pt: $(2, 4, 6)$

Direction vector: $(3,4,8)-(2,4,6)$

$= \langle 1,0,2 \rangle$

$(2,4,6) + t\langle 1,0,2 \rangle$

$= (2+t)i + (4)j + (6+2t)K$

$t \in \mathbb{R}$

Planes

Equation for a plane

start at one

- base point (x_0, y_0, z_0) (constants) on the plane.
- Use a perpendicular (normal) vector $\langle A, B, C \rangle$ to find other pts (x, y, z) (variables) on the plane.

$\langle A, B, C \rangle \cdot [\langle x, y, z \rangle - \langle x_0, y_0, z_0 \rangle] = 0$

$A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$

$Ax + By + Cz = Ax_0 + By_0 + Cz_0$

Ex) Find the plane $\perp$ to $\langle 1, 0, 1 \rangle$ to $\langle 2, 4, 6 \rangle$

$2x + 4y + 6z = 1(2) + 0(4) + 1(6)$

$2x + 4y + 6z = 2 + 6$

$2x + 4y + 6z = 8$

$x + 2y + 3z = 4$

Ex) Find an equation for the plane through $P_0 = (-3, 0, 7)$ $\perp$ to $n = 5\vec{i} + 2\vec{j} + \vec{k}$.

$5x + 2y + z = (-3)(5) + 2(0) + 7(1)$

$5x+2y+z=-8$

Ex] Find an equation for the plane through

$A(0,0,1)$, $B(2,0,0)$, & $C(0,3,0)$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} i & j & k \\ 2 & 0 & -1 \\ 0 & 3 & -1 \end{vmatrix} = \langle 3, 2, 6 \rangle$$

$\vec{AB} = \langle 2, 0, -1 \rangle$

$\vec{AC} = \langle 0, 3, -1 \rangle$

$\perp$ to the pl

$pt = (0,0,1)$, $\langle 3, 2, 6 \rangle$

$3x+2y+6z=6$